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THESIS

A MODEL FOR NARF SUPPLY SUPPORT WHICH INCLUDES BOTH ON-SITE SPARES AND SCHEDULED DELIVERY

by

Vance D. Berry, Jr.

March 1984

Thesis Advisor: A.W. McMasters

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A Model for NARF Supply Support Which Includes Both On-Site Spares and Scheduled Delivery

by

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Lieutenant, United States Navy
B.S., United States Naval Academy, 1978

Submitted in partial fulfillment of the
requirements for the degree of

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I. INTRODUCTION

A. BACKGROUND

In 1978, the Department of Defense Material Distribution System Study recommended the consolidation of wholesale supply support between the Naval Supply Centers (NSC) at Norfolk, San Diego, and Oakland, and their local Naval Air Stations.

As part of a study of how to improve the local area material distribution at Oakland and San Diego, McMasters [Ref. 1: pp. 1-6] addressed the problem of providing supply support for the Naval Air Rework Facility in each air station. In that report, he suggested several support methods. One was to provide on-site inventories at the NARF. The advantages of such a plan would be quicker response time to the customer, reduced transportation costs, and reduced customer delay costs. The costs of maintaining a separate inventory, however, would be a disadvantage. Another support method was through direct delivery by the NSC, with no on-site spares. A third possibility was a combination of the two methods; on-site inventories at the NARF with direct deliveries from the NSC when demand exceeds on-site inventories. The optimum solution of such a problem should be a trade off between customer needs, delivery costs, transportation costs, and costs which result from maintaining a separate inventory.

In the process of modelling these alternatives, McMasters [Ref. 2: pp. 4-14] developed a model for determining the depth of repair parts to be stocked on-site using a time independent delay cost. This model also addressed costs of establishing an inventory on site, penalty costs for being out of stock at some time before the end of a fixed time period, and surplus costs for having units remaining at the end of the fixed time period. He also modelled three alternative methods of providing direct delivery support [Ref. 1: pp. 7-42]. The costs included in these models were delivery costs, time dependent customer delay costs, and time independent customer delay costs. No attempt was made to combine the models.

B. PURPOSE

This thesis will develop a model which combines both an inventory system for on-site spares and a direct delivery model. The direct delivery model to be used is the scheduled delivery model which provides for delivery of all demands at the end of every N periods if there is at least one demand during those N periods. The cost of a delay for this model will include a time dependent as well as time independent costs. The resulting formulas will be subjected to parametric analysis for determination of optimum methods. The objective of the model will be to determine the on-site quantity which minimizes the total costs over N periods.

C. THESIS ORGANIZATION

Chapter I gives the background of the problem, the purpose, and the organization of this thesis. Chapter II establishes the probabilistic basis of the time dependent delay costs incurred by the NARF waiting for delivery from the NSC, given that there are N periods of time between deliveries. This result is conditioned by the probability that the on-site items are consumed before the end of N periods. Each element in the total cost equation is also examined, and the associated expected values are developed. Chapter III analyzes the model developed in Chapter II and seeks to identify the optimum number of on-site spares for various values of important parameters. Chapter IV compares the total cost results using no on-site spares and scheduled delivery as determined by Davidson [Ref. 3: pp. 18-35] to the optimal results of Chapter III. Chapter V summarizes the finding of Chapters III and IV and makes recommendations as the possible uses of the results and areas of further research.

II. MODEL DEVELOPMENT

This chapter develops the combination of an on-site inventory system and the scheduled direct delivery models of References 1 and 2.

A. ASSUMPTIONS

The work load schedule at a NARF is such that inductions of a component into overhaul occur at a specified constant rate during a quarter. As a consequence, the time between inductions is also a specified constant and can be used as a convenient measure of time. In this thesis it will be referred to as a time "period."

Under scheduled deliveries without an on-site inventory system, the NSC's truck makes a delivery at the end of every N periods if there is at least one demand during the N periods. If, however, stock exists on-site then it can be used to fill demand and thus reduce the chance that the truck will need to make a delivery during N periods.

In the development of the combined model the following assumptions will be made:

1. The time between potential demands is the time between inductions of a component.
2. At most one unit of a given repair part is required by each aircraft component undergoing rework at a NARF.

3. The probability that the component requires the repair part is p.

4. There are Y spare units of a given part on-site that will be consumed before requisitioning additional units from the supply center.

5. Delivery will be made at the end of N periods if at least Y+1 units are demanded during N periods.

As an example of the process, consider Figure 2.1. There are six inductions per quarter, and Y = 1. The probability that a part will require replacement is 0.4. The first period induction requires a spare which is satisfied by the on-site spare. The induction of periods 2 and 3 do not require spares. Another spare is required in period four and, since no stock is available, the part is requisitioned from the local NSC. An additional spare is required in period six creating two shortages during the quarter. Since delivery is scheduled for the end of the quarter, delivery of both parts will be made to the NARF. The process will continue during the next quarter with the assumption of Y spares on-site.

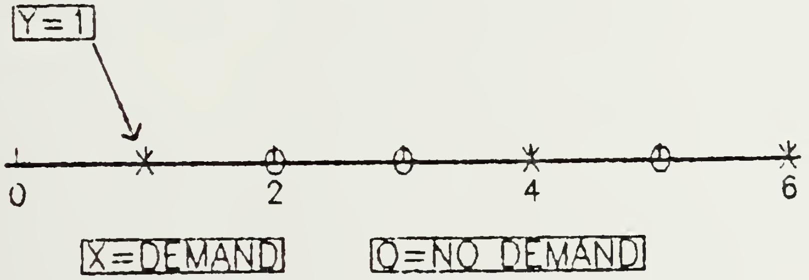

Figure 2.1 Example of Schedule Delivery with On-site Spares with N = 6, Y = 1, and p = 0.1.

Cost elements of the model will include Ct, the total round-trip costs of a delivery. These include the costs of a truck and a driver from the time it starts loading at the NSC to the time it returns to the NSC from its delivery. There also will be special handling costs, Ch, and processing costs, Cp, incident to the expense of establishing a unit in inventory on-site at the NARF.

Delay costs are costs incurred at the NARF as a consequence of not having a needed part the instant it is required. Two elements will compose the delay costs. One is S, which is the cost associated with putting a component aside. The second element is Cd, a time dependent delay cost, which will represent the delay cost per demanded unit per period. This element may include labor cost due to work stoppage, inventory holding costs, and cost associated with the non-availability of a repaired component to a fleet unit.

B. DETERMINISTIC DEMAND

If demand from a customer occurs with certainty once every time period, the demand is deterministic. If a truck is dispatched every time a demand is received, the cost per unit delivered is Ct. If k demanded units are allowed to accumulate before delivery then the cost per unit delivered is Ct/k. If the truck capacity is n units, then the delivery cost per unit is minimized by waiting until the truck is full.

If $Y \geq N$, there will be no delays as the on-site spares will satisfy all demand requirements of the system. However,

if Y < N, and if we define k as (N-Y), then as k units accumulate, the NARF experiences delay costs from lost production while awaiting parts. If we refer to the cost of delay of one time period as Cd, and if the NARF must wait until k demands are accumulated before the supply center makes a delivery, the total delay cost incurred by the NARF is

$$\frac{Cdk(k-1)}{2} .$$

This formula is based on the deterministic demand of one unit per time period. If a unit is demanded during the first period after a delivery then there will be a delay of k-1 periods until the arrival of the spare. Likewise, a demand on the second time period after a delivery will have a delay of k-2 time periods. So the total delay time from delivery to delivery will be

$$(k-1) + (k-2) + \ldots + 1 + 0 = \frac{k(k-1)}{2} ,$$

which can be rewritten, with the addition of the Cd term, as

$$\frac{Cdk(k-1)}{2} . \qquad (1)$$

C. RANDOM DEMAND

Equation (1) assumes that a demand is made every period. This is analogous to a part that is replaced in every component

undergoing an overhaul. If the repair part has a probability, p, $0 < p \leq 1$, that it will be replaced in the component during overhaul, we then must consider a case where delay costs are a function of the probability of demand, as well as N and Y. Our goal is how to determine the expected costs per period associated with the system.

The first stage of development will determine the expected total delay time given that there are Y repair items on-site. In order for a delay to occur, there must be Y+1 units demanded before the end of the Nth time period. If we consider the process of replacement of a part of a component as an independent Bernoulli trial with probability p, then we must determine the distribution of the number of independent Bernoulli trials required for Y demands during N periods. This process is described by the negative binomial probability distribution.

Let n be the number of trials (periods of demand), necessary to observe Y demands. Clearly, the range of n is $R = Y, Y+1, \ldots, \infty$. The negative binomial probability function for a given Y is

$$P(n;Y) = \binom{n-1}{Y-1} p^{Y} (1-p)^{n-Y} ; \qquad (2)$$

where

$$n \in \{Y, Y+1, \ldots, \infty\} .$$

In our problem, (2) represents the probability that it will take exactly n periods for a total demand of Y to occur. As an example of Equation (2), let us say there are a total of 6 time periods, (N = 6), and the number of units on hand, Y, is one. The probability, p, of a demand during each time period is 0.4. The figure below illustrates the probability that the Yth demand first occurs on each of the periods one through six. Since we have restricted the range of n to have an upper bound of N = 6, the sum of the probabilities will be less than one.

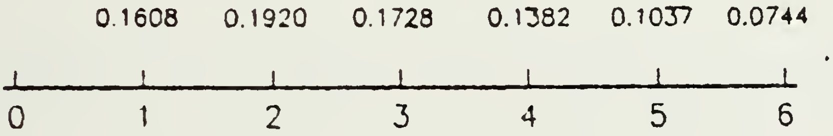

Figure 2.2 Distribution of Negative Binomial with n = 1, N = 6, and p = 0.4.

Given that it takes n periods for the Yth demand to occur, there will be exactly (N-n) time periods before a delivery is made if there is an additional demand before the end of the N periods. In determining the expected total delay cost for scheduled delivery with Y items on site where there are (N-n) periods remaining for a delay, it is useful to review the steps for obtaining the expected delay costs for N periods as presented in Reference 1.

Delays are a function of the number of configurations that demands can take in N periods. The total number of configurations where exactly x demands occur is expressed by

$$n_x = \binom{N}{x} . \qquad (3)$$

The total number of configurations which can occur with at least one demand is

$$n = \sum_{x=1}^{N} n_x = \sum_{x=1}^{N} \binom{N}{x} = 2^{N-1} \qquad (4)$$

To determine the expected total delay associated with the n configurations given by (3), we first consider only those configurations having exactly x demands where $x \geq 1$. The probability of each such configuration is

$$P(x;N) = p^x (1-p)^{N-x} . \qquad (5)$$

The number of configurations having a demand in period $1 < j < N$ is

$$m = \binom{N-1}{n-1} . \qquad (6)$$

It is significant that m is independent of j. Those demands occurring in period j will have to wait until period

N for delivery and hence each must wait N-j periods. The total of all delays for those configurations having x demands is

$$TD(x,N) = \sum_{j=1}^{N} \binom{N-1}{x-1}(N-j)$$

$$= \binom{N-1}{x-1} \sum_{j=1}^{N} (N-j)$$

$$TD(x,N) = \frac{N(N-1)}{2} \binom{N-1}{x-1} . \qquad (7)$$

From (5) and (7) we can find the expected total delays over all x values:

$$ETD(N) = \sum_{x=1}^{N} TD(x,N)P(x,N)$$

$$= \sum_{x=1}^{N} \frac{N(N-1)}{2} \binom{N-1}{x-1} p^{x} (1-p)^{N-x}$$

$$ETD(N) = \frac{N(N-1)}{2} \sum_{x=1}^{N} \binom{N-1}{x-1} p^{x} (1-p)^{N-x} . \qquad (8)$$

By factoring the summed terms,

$$ETD(N) = \frac{N(N-1)p}{2} \sum_{x=1}^{N} \binom{N-1}{x-1} p^{x-1} (1-p)^{N-x} .$$

The summation term is now equal to one and our expression reduces to

$$ETD(N) = \frac{N(N-1)p}{2} . \qquad (9)$$

Now, given that the last of the Y repair items is demanded in the nth time period, it easily follows that there are (N-n) time periods remaining in which demands may occur, the expected total delays can be expressed as

$$TD(N-n) = \frac{(N-n)[(N-n)-1]p}{2} . \qquad (10)$$

However, since there is a probability associated with n time periods being required for the Yth demand to occur, Equation (10) must be multiplied by that probability, given in (2), and the result summed over n to get the expected total delay over N time periods, given an on-site inventory of Y. The expected total delay cost is:

$$ETDC(N;Y) = Cd \sum_{n=Y}^{N} \binom{n-1}{Y-1} p^{Y} (1-p)^{n-Y} \left[\frac{(N-n)((N-n)-1)p}{2}\right]. \qquad (11)$$

Upon examination of Equation (11), we see that for a fixed N, the expected total delay cost is monotonically decreasing with an increasing Y. Furthermore, ETDC(N,Y) → 0 as Y approaches (N-1).

D. COSTS INCURRED WITH THE ESTABLISHMENT OF Y ITEMS ON-SITE

Reference 2 identified two costs, special processing and holding, associated with locating Y items on-site. Special processing costs, Cp, are those paperwork and processing costs charged when placing an item in an on-site store. If a quantity of Y items are placed into an on-site store the total processing costs will be CpY.

The space required to store on-site units must be large enough to accommodate all Y units. In addition, the cost of that space can be expected to be constant even during periods when the number of items on-hand are less than the quantity Y. The total holding costs will therefore be ChY.

E. TIME INDEPENDENT DELAY COSTS

The cost per unit associated with putting a component aside when all Y spares are expended before delivery at time N will be denoted as S. This cost is assessed only at the time of the demand and is not therefore time dependent. It can include the cost of placing a component in storage and documentation of the status of repair and requisitions. To find the associated expected delay cost we must first find the expected number of components that will suffer delays because of lack of repair parts when required.

If x is the number of units demanded in N periods, then when $x > Y$ the number of components set aside will be the difference $(x-Y)$. The probability distribution that describes the total number of demands for a series of independent

Bernoulli trials is the binomial distribution. Therefore the expected time independent delay costs are

$$S \sum_{x=Y+1}^{N-1} (x-Y) \binom{N}{x} p^{x} (1-p)^{N-x} . \qquad (12)$$

Note that the upper bound of the summation is N-1, for if a demand occurs in the Nth time period it is assumed to be filled immediately and no delay occurs.

F. SURPLUS COSTS

Reference 2 also examines surplus costs. The unit cost for having a surplus of items in on-site inventory at the end of N time periods is kC where C is the unit cost and k is a factor which may be greater than 1.0. If $x < Y$, then the cost of surplus in N time periods is kC(Y-x). The expected total surplus cost may be found by the same method as the time independent delay cost.

$$kC \sum_{x=0}^{Y-1} \binom{N}{x} p^{x} (1-p)^{N-x} \qquad (13)$$

G. DELIVERY COSTS

When considering delivery cost, the cost associated with making a delivery at time N will be denoted by the term Ct. Since a delivery only can occur if the number of demands x exceeds the on-site spares Y in N time periods, the probability

of x exceeding Y will be the sum of the binomial probabilities for x from Y to N. Therefore the expected delivery cost will be:

$$Ct \sum_{x=Y+1}^{N} \binom{N}{x} p^{x} (1-p)^{N-x} . \tag{14}$$

H. EXPECTED TOTAL COSTS WITH A FIXED N

The expected total cost over a fixed total number of periods N is found by summing all of the expected cost elements described above.

$$\begin{aligned} ETC(Y;N) \quad = \quad & \left(\begin{matrix}\text{Special Handling}\\ \text{+ Processing Costs}\end{matrix}\right) + \left(\begin{matrix}\text{Surplus}\\ \text{Costs}\end{matrix}\right) \\ & + \left(\begin{matrix}\text{Time Independent}\\ \text{Delay Costs}\end{matrix}\right) + \left(\begin{matrix}\text{Delivery}\\ \text{Costs}\end{matrix}\right) \\ & + \left(\begin{matrix}\text{Time Dependent}\\ \text{Delay Costs}\end{matrix}\right) ; \end{aligned} \tag{15}$$

$$\begin{aligned} ETC(Y;N) \quad = \quad & (Cp + Ch)Y + kC \sum_{x=0}^{Y-1} \binom{N}{x} p^{x} (1-p)^{N-x} \\ & + S \sum_{x=Y+1}^{N} (x-Y) \binom{N}{x} p^{x} (1-p)^{N-x} \\ & + Ct \sum_{x=Y+1}^{N} \binom{N}{x} p^{x} (1-p)^{N-x} \\ & + Cd \sum_{n=Y}^{N} \binom{n-1}{Y-1} p^{n} (1-p)^{n-Y} \left[\frac{(N-n)((N-n)-1)p}{2}\right] . \end{aligned} \tag{16}$$

This chapter has developed a formula for expected total costs for a system which combines an inventory system for on-site spares and a direct delivery model. Equation (16) will next be analyzed in an attempt to determine the optimal values of the decision variables Y and N.

III. OPTIMIZATION ANALYSIS

Since N and Y can take on only discrete values, the use of finite differences is appropriate for determining their optimal values. However, since optimization formulas for N and Y based on finite differences were as complex as the original cost equation (16), an APL program was written which numerically determined the expected total cost for a range of N and Y values. The program is included in Appendix A. The results were then plotted and optimal values were determined by examination of the graphical results.

In order for this analysis to be comparable with Reference 1 and Davdison's analysis in Reference 3, the following values were assumed for the cost terms:

Time dependent delay cost (Cd)	\$50 per unit per period
Time indepedendent delay cost (S)	\$20 per unit
Special handling cost (Ch)	\$0.01 per unit
Special processing cost (Cp)	\$1.00 per unit
Surplus cost (kC)	\$250 per unit

Since the total number of time periods and probability of demand are likely to be fixed in practice, the analysis uses fixed values of N and p and varies Y.

Figure 3.1 provides a look at the total cost of a scheduled delivery scheme with fixed N between from 10 and 50 and

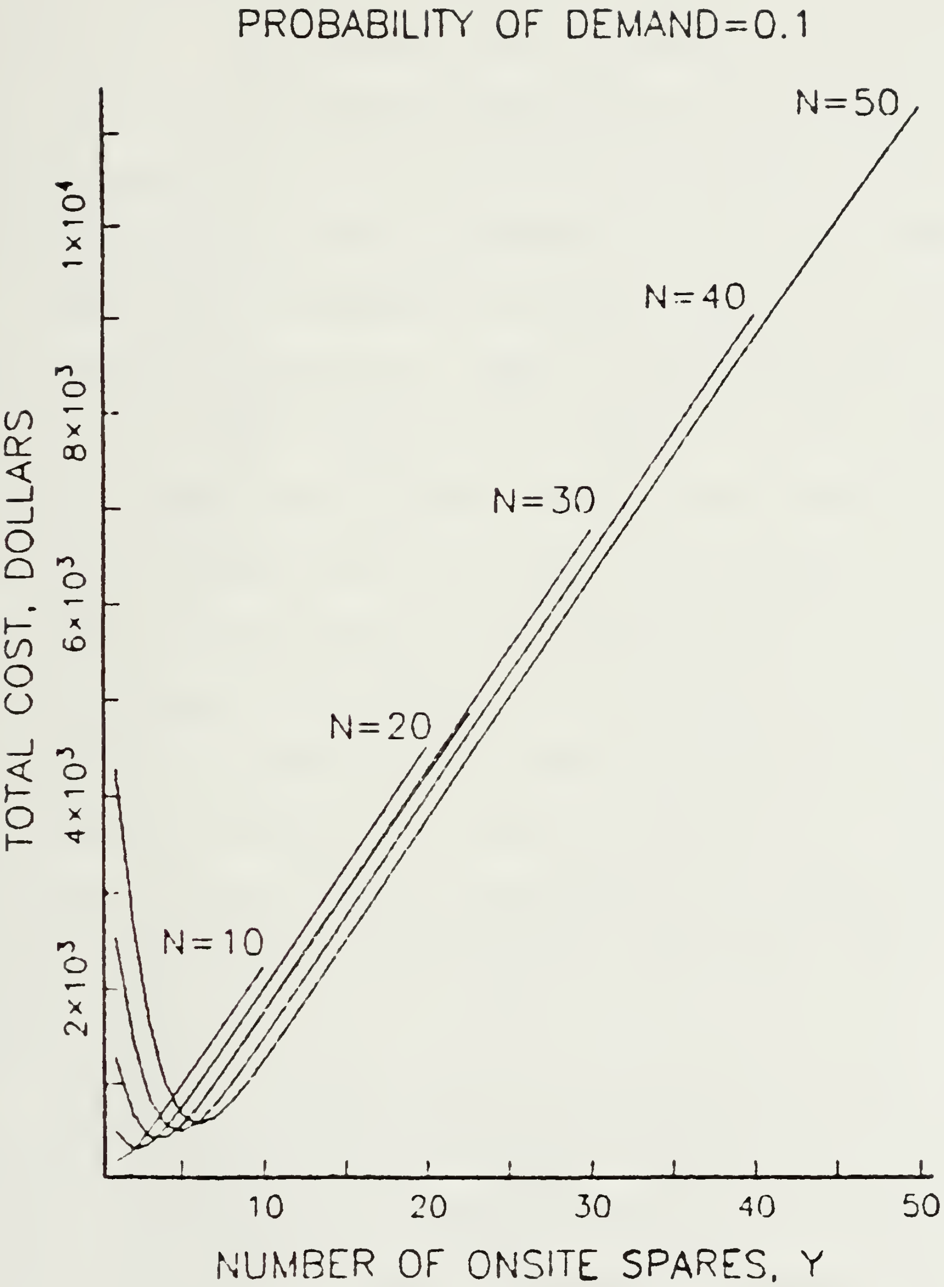

Figure 3.1 Total Cost as a Function of On-site Spares with Varying Values of N.

$p = 0.1$. Although the total cost values are discrete, a curve is drawn through those points for clarity. The N values were chosen to illustrate the general shape of the curve for a range of N. This figure also illustrates the convexity of the curves when costs are presented on a linear scale. Subsequent figures will use a log scale for total costs to facilitate comparison between parameters and may not appear convex.

Note that the number of on-site spares, Y, was limited to a maximum value of N. This is consistent with the development of the model because if Y exceeded N then the number of on-site spares would always exceed total demand and this situation clearly would not lead to an optimal solution. The cost curves illustrate this observation as the total cost approaches its maximum as the value of Y approaches N. As would be expected with a small probability of demand, the optimum value of Y is small relative to N. The time dependent delay costs are small with low demand and the surplus and special handling and processing costs will increase with a large Y.

Figure 3.2 illustrates the components of the expected total cost curve as described by Equation (16) for the case of $N = 50$. Figure 3.2 shows that the major components of the total cost curve are the time dependent delay cost, Cd, and the surplus cost, kC. For the given parameters, the delivery cost, Ct, and the time independent delay cost, S, do not have a significant impact on total cost. As expected, both of these terms strictly decrease as Y increases. The special handling

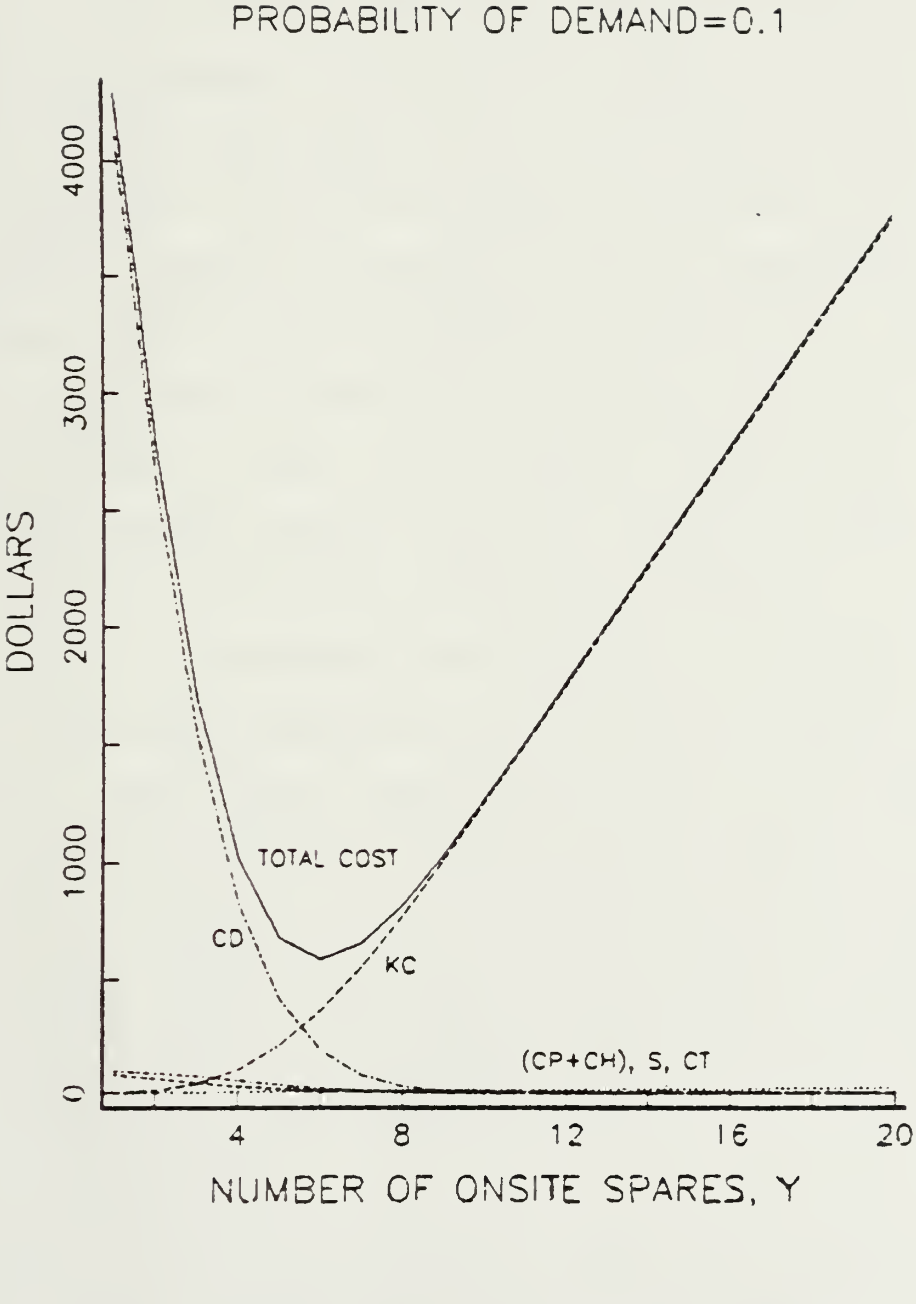

Figure 3.2 Components of the Total Cost with N = 50 and p = 0.1.

and processing cost term, increases linearly with increasing Y but due to the value assigned to (Cp + Ch), the term has little impact on total cost.

Figure 3.3 presents total cost curves on a log scale for varying Y with a fixed N and three different values for the probability of demand, p. The first graph in 3.3 represents the same situation as Figure 3.1 but with costs presented with a log scale. As can be seen from the graphs, as the probability of demand increases so does the number of on-site spares required for optimality for the same N. This is due to the increase in both delay cost terms when the demand exceeds Y, which is more likely with an increasing p. These costs will outweigh surplus costs for excess Y, which are less likely with increased probability of demand.

Figure 3.4 illustrates how the optimal value of Y varies with N. The figure emphasizes the discrete values of N and optimum Y. The distinct break points for $p = 0.1$ become less pronounced as p increases, becoming nearly linear as p approaches 1.0. When $p = 1.0$, $Y = N$, both delay cost and surplus costs will be zero, and the only costs with a positive value will be the Ch and Cp terms, which are linear in Y. This result should apply whenever Ch and Cp are much less than Cd, S, and kC.

Figure 3.5 shows how the total costs vary over a range of probabilities of demand for a fixed N and selected Y values. These graphs also show that for a given N value, systems

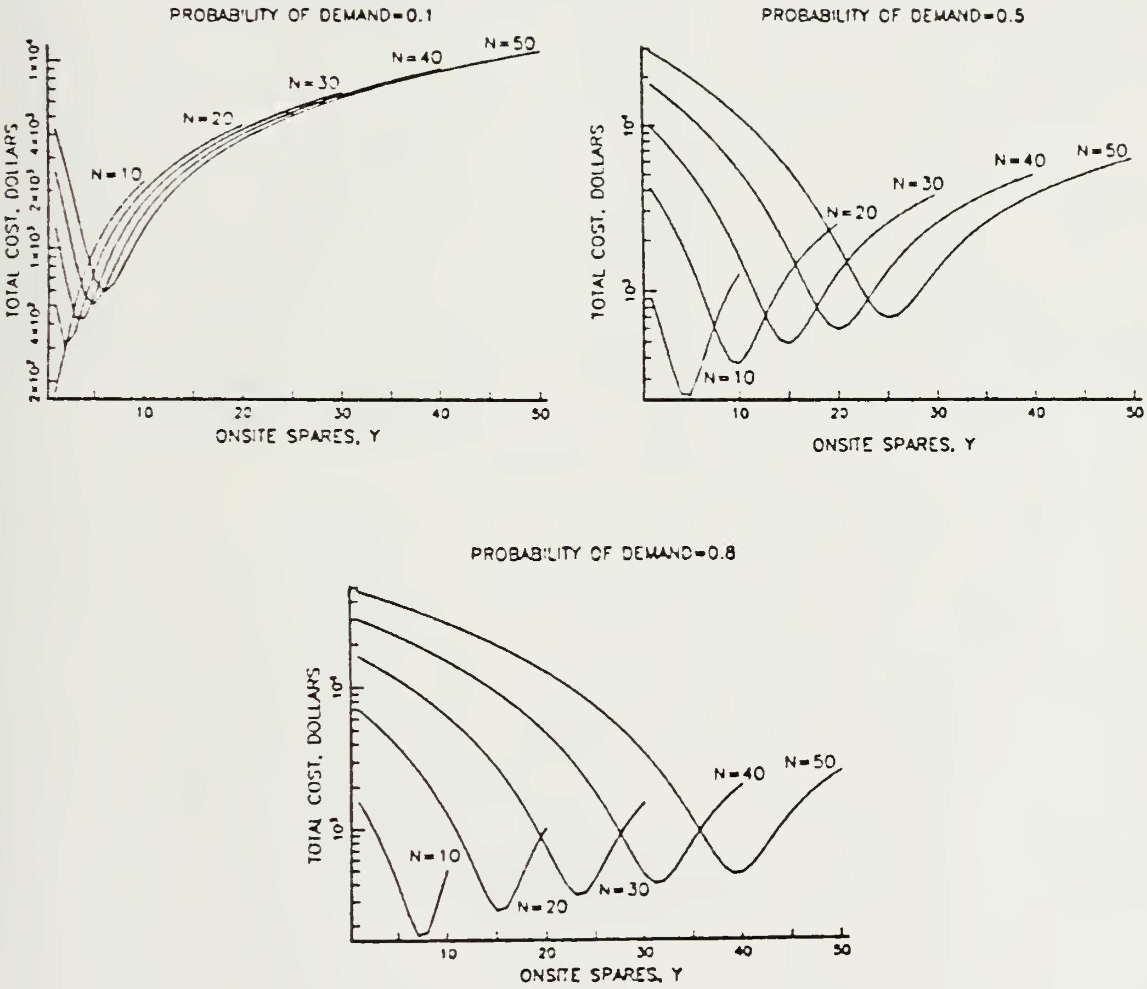

Figure 3.3 Total Cost as a Function of the Number of On-site Spares for Varying Values of N and p = 0.1, 0.5, and 0.8.

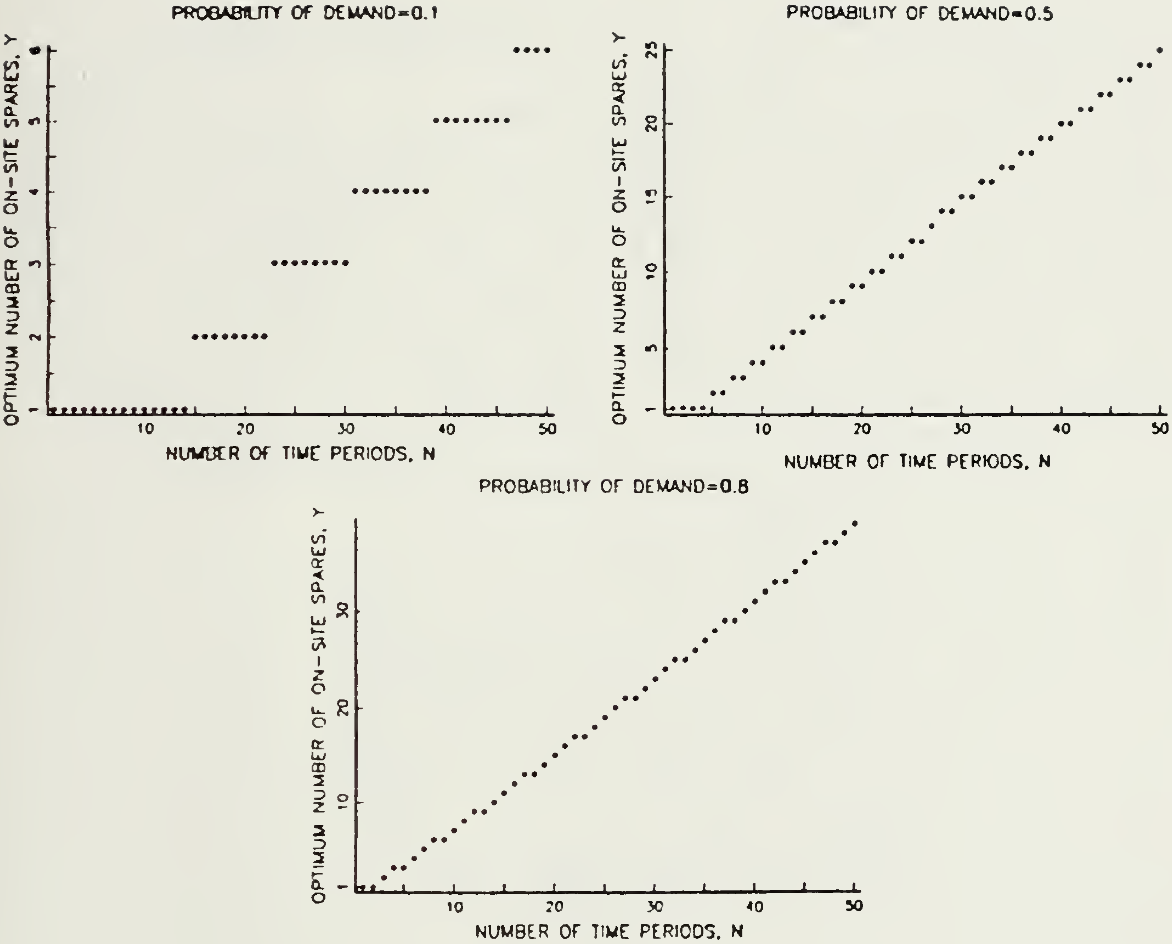

Figure 3.4 The Optimal Number of On-site Spares Y as a Function of the Number of Time Periods N for p = 0.1, 0.5, and 0.8.

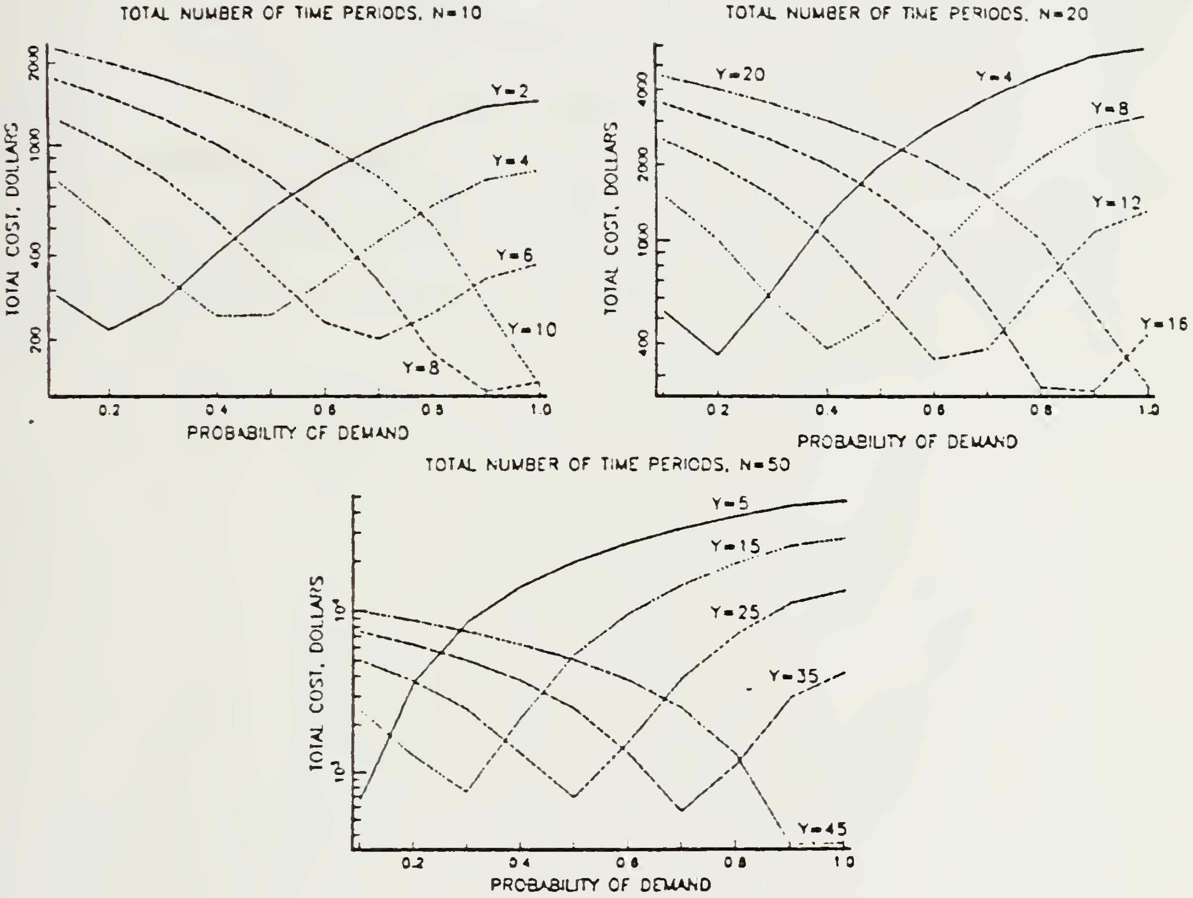

Figure 3.5 Total Cost as a Function of the Probability of Demand for Varying Values of Y with N = 10, 20, and 50.

with low probability of demand require smaller on-site stores to minimize total cost, as was noted earlier.

Figure 3.6 shows the relationship between optimal Y and the probability of demand for three values of N. Though p is continuous, for the purposes of illustration, the probabilities of demand are varied from 0.1 to 1.0 with increments of 0.1. Again, as probability of demand approaches 1.0, the optimum value of Y approaches N. For the given set of cost parameters these plots show that optimal Y is approximately equal to the expected demand pN. The relationship between Y and pN becomes more nearly linear with increasing N.

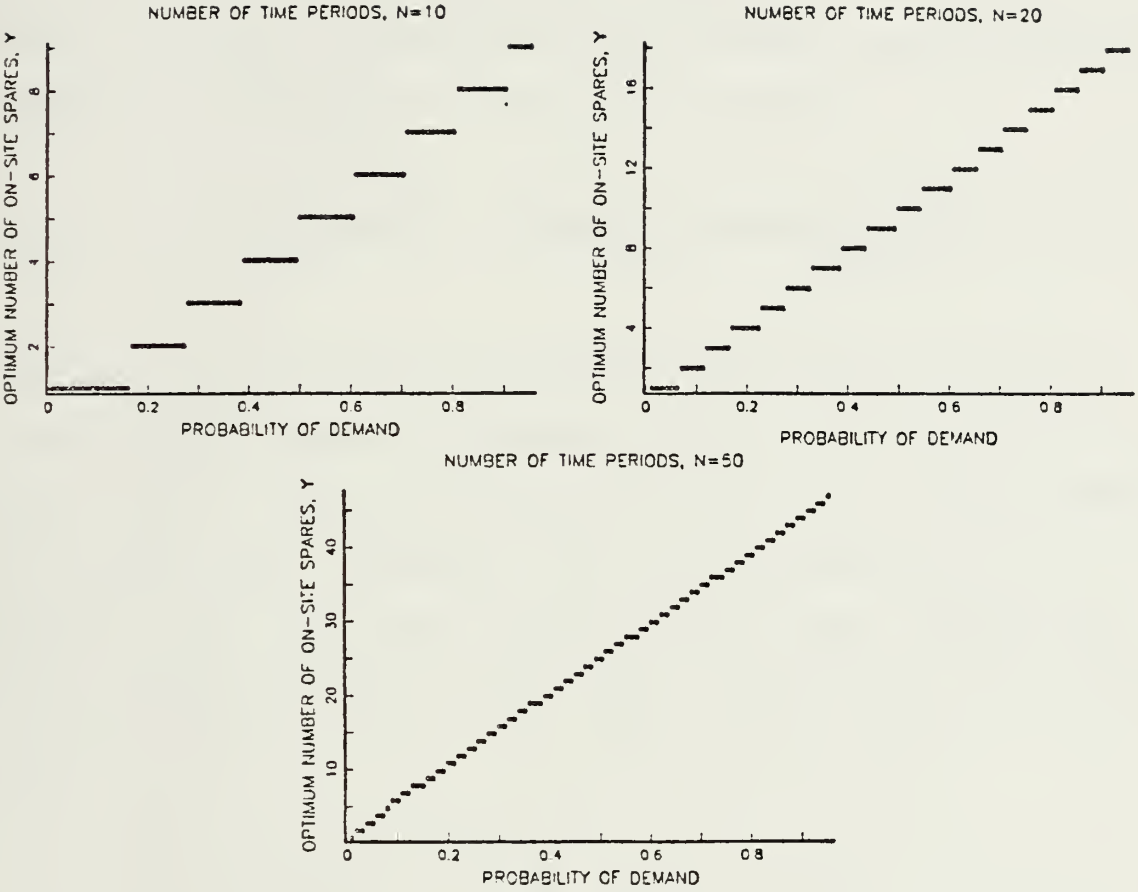

Figure 3.6 The Optimal Number of On-site Spares Y as a Function of the Probability of Demand for N = 10, 20, and 50.

IV. COMPARISON TO A SCHEDULED DELIVERY SYSTEM WITHOUT ON-SITE SPARES

Davidson in Reference 3 performed a parametric analysis of costs of a scheduled delivery system with no on-site spares. The cost equation used by Davidson was developed by McMasters in Reference 1 and is of the form,

$$\mathrm{ECP}(N) = \left[\frac{Ct[1-(1-p)^N]}{N} + \frac{Cd(N-1)p}{2}\right]\left[\frac{-\ln[1-(1-p)^N]}{(1-p)^N}\right] . \quad (17)$$

Equation (17) describes the expected costs per period and there are only two cost elements, delay cost and delivery cost. For a comparison between (16) and (17), an adjustment to (17) was made. Equation (17) was multiplied by N to obtain a total cost value over N and the time independent delay cost term from (16) was added. The result is Equation (18).

$$\mathrm{ETC}(N) = Cd\left[\frac{N(N-1)p}{2}\right] + S\sum_{x=1}^{N}\binom{N}{x}p^x(1-p)^{N-x}$$

$$+ Ct\sum_{x=1}^{N}\binom{N}{x}p^x(1-p)^{N-x} . \quad (18)$$

The expected total costs of both (16) and (18) were numerically evaluated using the same values for Cd, S, and Ct as the examples presented in Chapter III. The Cp and

Ch terms in (16) were also the same as those used in Chapter III.

Table 1a presents the results of the comparison when the probability of demand is 0.1. It displays the total expected cost for the two models. Model 1 corresponds to Equation (16) with an optimal number of on-site spares, and Model 2 corresponds to Equation (18) for the same N values. Tables 1b and 1c present the results for probability of demand of 0.5 and 0.8, respectively. As Table 1 shows, the model employing an optimal number of on-site spares has a smaller expected total cost than a system not employing on-site spares and the savings provided by an on-site system can be significant. The total expected costs with no on-site spares start substantially higher for all three probability values, and increase faster with increasing N than they do for the on-site spares model. This difference can be explained by the impact of the Cd term. In the case of no on-site spares, the Cd term,

$$\frac{CdN(N-1)p}{2},$$

increases at a geometric rate with an increasing N, whereas in the on-site model, cost savings are achieved by delaying the application of the Cd term.

A key question in the future comparison of these two models will be: "At what point do the costs of the implementation of an on-site system no longer make it preferable to a system without on-site spares?"

TABLE I

Comparison of Model 1 and Model 2 with p = 0.1, 0.5, 0.8

Table 1a

Probability of Demand = 0.1

Number of Periods	Total Expected Costs Model 1	Total Expected Costs Model 2
10	172.29	310.13
20	305.65	1077.84
30	432.70	2330.76
40	506.33	4078.52
50	587.53	6324.48

Table 1b

Probability of Demand = 0.5

Number of Periods	Total Expected Costs Model 1	Total Expected Costs Model 2
10	240.95	1324.70
20	371.01	5049.50
30	485.14	11275.00
40	591.55	20000.00
50	693.14	31225.00

Table 1c

Probability of Demand = 0.8

Number of Periods	Total Expected Costs Model 1	Total Expected Costs Model 2
10	169.41	463.5
20	254.17	8015.39
30	328.71	17979.23
40	397.93	31939.89
50	463.7	49899.48

Upon examination of Equation (16), we find that the special processing and handling costs, (Cp + Ch), and the time independent delay costs, S, can be examined to answer this question. The condition of indifference between the two methods is described by equating (16) and (18), or,

$$ETC(Y^*;N) = ETC(N)$$

where Y* is the value of Y that minimizes total expected costs.

Both the (Cp + Ch) term and the S term will affect the value of Y* as they are varied. The effect of the (Cp + Ch) term is suggested as follows. Suppose that we represent Equation (16) by

$$TVC = f_1(Y) + (Cp + Ch)Y , \qquad (19)$$

where f(Y) represents all other cost elements of the equation excluding (Cp + Ch). If Equation (19) were continuous in Y we could take the derivative with respect to Y and set the result equal to zero, or,

$$\frac{dTVC}{dY} = \frac{df_1(Y)}{dY} + Cp + Ch = 0 . \qquad (20)$$

Rearranging terms gives,

$$\frac{df_1(Y)}{dY} = -(Cp + Ch) \ , \qquad (21)$$

and Y* could be determined from Equation (21). Equation (21) clearly shows that a change in Cp + Ch will affect the value of Y*.

The determination of the breakeven point for the (Cp + Ch) term can be found by fixing the value of all parameters with the exception of (Cp + Ch) and determining Y*. The expected total cost of (16) with Y* and N is compared to (18) with the same N. The value of (Cp + Ch) is varied and Y is recomputed and again the total costs of the two equations are compared. The process continues until the value of (Cp + Ch) is found that makes the expected total cost of (16) and (18) equal.

The effect of a change in S, the time independent delay costs, with respect to Y* is similar. Any change in S will change the shape of the total expected cost curve which in turn will affect Y*. The procedure described above can be used to find the breakeven point for S.

Variation of the Ct and Cd values will have no effect on the preference of the model described by (16) over that described by (18). Upon examination of the Ct and the Cd terms in Equation (16), it is easy to see that the expected costs of these two component terms are maximized when Y = 0. Clearly at that point, the terms in Equation (16) are identical to their analog in (18). Therefore, a change in either Ct or Cd will not affect the preference of (16) over (18).

V. CONCLUSION AND RECOMMENDATIONS

A. CONCLUSION

This thesis addressed the problem of supply support for a NARF by the local NSC. A model using a combination of scheduled deliveries from the NSC and on-site spares at the NARF was developed. The optimal results of the model were obtained for several values of the probability of demand. These results were also compared to an earlier model for scheduled delivery without on-site spares, developed in References 1 and 3.

The most noteworthy point of this thesis is that the model that uses on-site spares was found to have significantly lower total expected costs than the model that does not use on-site spares. Although several of the cost values chosen were hypothetical since data for the delay costs does not exist, they do serve to provide a relative comparison. If the actual values of the cost parameters can be established, the analysis can be repeated easily to determine how much better performance can be obtained with a combined on-site spares/scheduled delivery model than with the pure scheduled delivery model, and the optimal number of on-site spares.

Also of note is the utility of Figures 4 and 6 in the management of a supply support system. If such a set of figures were available for actual cost values and changes did

occur in the probability of demand for an item, or the total number of time periods, the new optimal number of on-site spares could be readily determined from such figures.

B. RECOMMENDATIONS

McMasters in Reference 2 proposed two other delivery methods. One method assumed that a delivery is delayed until some fixed number of units of an item have been demanded. Delivery is then assumed to take place as soon as the last demand occurs. The second method starts by counting time from when the first demand occurs after the truck has returned from the NARF and is ready for further deliveries. Delivery is made M-1 periods after the first demand. These two models should also be evaluated since they might provide lower costs than the current scheduled delivery model when combined with the on-site stocking.

However, comparison between the three combinations of on-site delivery when those inventories are depleted will require the evaluation of total variable costs per time period. Thus, a renewal argument will be needed. The basis for that argument has already been established by Reference 1. The first step of such an analysis is to relax the constraint imposed in Chapter II that $n < N$. In fact, n has an infinite upper bound as Equation (2) has indicated. From Equation (2) the probability of no delivery in the first N periods is

$$\sum_{n=N}^{\infty} p(n;Y) \ , \qquad (22)$$

and the probability of no delay during the first delivery period is

$$\sum_{n=N-1}^{\infty} p(n;Y) \ . \qquad (23)$$

These two probability statements can be subdivided into a sequence of many periods of length N. We can then consider the possibilities of Y not being used up in N periods, 2N periods, etc., and extend the model of Chapter II to cover those mutually exclusive alternatives.

Finally, an aspect which should be considered in these two delivery methods is when the on-site spares should be replenished as this event constitutes a renewal. Perhaps the next delivery after Y has been depleted should include Y units in addition to the demands which have occurred since the first Y was depleted.

APPENDIX A

APL PROGRAM FOR NUMERICALLY EVALUATING EXPECTED TOTAL COSTS

```
     ∇ GTEST
[1]  ⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝
[2]  ⍝ THIS APL FUNCTION COMPUTES THE EXPECTED TOTAL COST OF A
[3]  ⍝ COMBINED ON-SITE SPARES/SCHEDULED DELIVERY SYSTEM.  THE TOTAL
[4]  ⍝ NUMBER OF PERIODS N. IS INPUT BY THE USER AND THE FUNCTION
[5]  ⍝ COMPUTES TOTAL COSTS FOR THE SYSTEM WITH THE NUMBER OF ON-SITE
[6]  ⍝ SPARES VARYING FROM 1 TO N.  THE RESULTING ARRAY IS THEN EXAMINED
[7]  ⍝ TO DETERMINE THE VALUE OF Y THAT YIELDS THE LEAST TOTAL COST.
[8]  ⍝ THE ARRAY OF Y VALUES WITH THEIR CORRESPONDING TOTAL COST ARE
[9]  ⍝ PRINTED ALONG WITH THE VALUE OF Y THAT MINIMIZES TOTAL COST.
[10] ⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝
[11] ⍝   VARIABLE NAMES ASSIGNED TO PARAMETERS
[12] ⍝   P1------------PROBABILITY OF DEMAND
[13] ⍝   CD------------TIME DEPENDENT DELAY COST
[14] ⍝   CP------------PROCESSING COST
[15] ⍝   CH------------HOLDING COST
[16] ⍝   S-------------TIME INDEPENDENT DELAY COST
[17] ⍝   KC------------SURPLUS PENALTY
[18] ⍝   CT------------DELIVERY COST
[19]
[20] ⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝
[21]
[22] ⍝ THE FUNCTION PROMPTS THE USERS FOR TOTAL NUMBER OF PERIOD.
[23]  'ENTER N'
[24]  N←⎕
[25]
[26] ⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝
[27]
[28] ⍝ PARAMETER INITIALIZATION
[29]  MAT1← 1 2 ⍴0
[30]  P1←0.5
[31]  CD←50
[32]  CP←1
[33]  CH←0.1
[34]  S←20
[35]  KC←250
[36]  CT←100
[37]
[38] ⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝
[39]
```

```
[40] ⍝ ECHO OF PARAMETER VALUES
[41]  'PROBABILITY OF DEMAND         ',⍕P1
[42]  'DELAY COST                    ',⍕CD
[43]  'PROCESSING COST               ',⍕CP
[44]  'HOLDING COST                  ',⍕CH
[45]  'SHORTAGE PENALTY              ',⍕S
[46]  'SURPLUS PENALTY               ',⍕KC
[47]  'DELIVERY COST                 ',⍕CT
[48]
[49] ⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝
[50]
[51] ⍝ COUNTER FOR VALUE OF Y INITIALIZED AND RANGE OF Y ESTABLISHED
[52] ⍝ AND PARAMETERS FOR BINOMIAL PROBABILITIES ARE INITIALIZED.
[53] L1:COUNTER←1
[54]  TIME←N
[55]  ONSITE←⍳TIME
[56]  PAR←TIME,P1
[57]
[58] ⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝
[59]
[60] ⍝ BEGIN ALGORITHM TO COMPUTE EXPECTED TOTAL COST
[61] ⍝ NEXT VALUE OF Y SELECTED
[62] L2:Y←ONSITE[COUNTER]
[63] ⍝ THE EXPECTED COST OF THE TIME INDEPENDENT DELAY COST IS COMPUTED
[64] ⍝ USING A FUNCTION THAT DETERMINES TOTAL EXPECTED DELAY AS DESCRIBED
[65] ⍝ IN EQUATION (16).
[66]  A1←CD×Y NEGBIN TIME
[67]
[68] ⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝
[69] ⍝ SPECIAL HANDLING AND PROCESSING COSTS ARE DETERMINED BY MULTIPLYING
[70] ⍝ (CP+CH) AND THE CURRENT VALUE OF Y.
[71]  A2←(CP+CH)×Y
```

```
[72]
[73] ⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝
[74]
[75] ⍝ THE TIME INDEPENDENT DELAY COSTS, SURPLUS COSTS, AND DELIVERY COSTS
[76] ⍝ ARE DETERMINED BY MULTIPLYING EACH PARAMETER WITH A FUNCTION
[77] ⍝ THAT EVALUATES THE EXPECTED NUMBER TIME PERIODS DELAYED, SURPLUS
[78] ⍝ UNITS, AND PROBABILITY OF A DELIVERY, RESPECTIVELY.
[79]  A3←S×('>',(⍕Y)) BINOMIAL3 PAR
[80]  A4←KC×('(',(⍕Y)) BINOMIAL2 PAR
[81]  A5←CT×('>',(⍕Y)) BINOMIAL PAR
[82]
[83] ⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝
[84]
[85] ⍝ THE SUM OF ALL TERMS IS COMPUTED AND THE RESULTED IS PLACED
[86] ⍝ IN AN ARRAY
[87]  TOTAL←A1+A2+A3+A4+A5
[88]  LINE1←Y,TOTAL
[89]  MAT1←MAT1,[1] LINE1
[90]
[91] ⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝
[92]
[93] ⍝ THE VALUE OF Y IS INCREASED AND CHECKED TO SEE IF IT EXCEEDS
[94] ⍝ THE TOTAL NUMBER OF PERIODS.
[95]  COUNTER←COUNTER+1
[96]  →(COUNTER≤TIME)/L2
[97]
[98] ⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝⍝
[99]
[100] ⍝ THE ARRAY IS PRINTED AND ANOTHER FUNCTION DETERMINES THE VALUE
[101] ⍝ OF Y THAT MINIMIZES EXPECTED TOTAL COSTS.
[102]  MAT1← 1 0 ↓MAT1
[103]  PRINT MAT1
[104]  →0
     ∇
```

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